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Technical Note

The onset of vortex instability in laminar forced convection flow through a horizontal porous channel

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Abstract

The onset of convective instability in a fluid-saturated porous layer between the two horizontal plates heated isothermally from below has been analyzed theoretically by using propagation theory. In the analysis the thermal dispersion coefficient is assumed to be proportional to the streamwise velocity. The results show that both inertia and thermal dispersion stabilize the system.

1. Introduction

The system considered here is a fluid-saturated porous layer with the fully-developed laminar flow of uniform superficial velocity U_0. In the fluid-saturated porous layer confined between the two horizontal plates of depth H, the temperature is uniform at T_i for the streamwise distance $X \leqslant 0$ and for $X > 0$ there is a step change in the bottom temperature to a higher value T_w. The thermal boundary-layer thickness Δ_T increases with increasing X. Under this condition the primary flow of steady, laminar forced convection prevails and for a high T_w the secondary motion of vortex rolls caused by buoyancy forces will set in at a certain streamwise position X_c.

In the present system the important parameters are the particle Reynolds number Re_d, the effective Prandtl number Pr, the Péclet numbers Pe, and the local Nusselt number Nu_X, the Darcy number Da, the Rayleigh number Ra, and the Darcy–Rayleigh number Ra_D defined as

$$\begin{aligned}
&Re_d = U_0 d/\nu, \quad Pr = \nu/\alpha_e, \\
&Pe_d = U_0 d/\alpha_e, \quad Pe_H = U_0 H/\alpha_e, \\
&Pe_X = U_0 X/\alpha_e, \quad Nu_X = q_w X/(k_e \Delta T), \\
&Da = K/H^2, \quad Ra = g\beta \Delta T H^3/(\alpha_e \nu), \\
&Ra_D = DaRa,
\end{aligned} \tag{1a–1i}$$

where $\Delta T = T_w - T_i$. Here q_w denotes the heat flux at the bottom boundary, α_e the effective thermal diffusivity, ν the kinematic viscosity of fluid, k_e the effective thermal conductivity, K the permeability, g the gravitational acceleration, and β the thermal expansivity.

Under the Darcy model Prats [1] analyzed the onset of mixed convection in the above system and reported that the buoyancy-driven convection sets in when Ra_D exceeds the value of $4\pi^2$. This critical value is the same as that to represent the onset of Horton–Rogers–Lapwood convection in an initially, quiescent fluid layer heated very slowly from below. For $Ra_D \gg 4\pi^2$, it becomes an important problem to predict the critical streamwise position wherefrom mixed convection starts. This stability problem will be analyzed by using propagation theory and the effect of thermal dispersion on stability will be examined in detail.

2. Basic state of primary forced convection

Fand et al. [2] and Seguin et al. [3] reported that Darcy's law is valid for the particle Reynolds number

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$Re_d \leqslant 2.3$, Forchheimer's equation is valid for $5 \leqslant Re_d \leqslant 80$, and fully developed turbulent flow exists when $Re_d > 120$–180. In the present study it is assumed that the superficial velocity U_0 is constant and under local thermal equilibrium between solid particles and fluid the effect of thermal dispersion on heat transfer is critical. Also, constant thermal dispersion and no effects of wall and tortuosity are assumed. A wide range of parameters affecting flow and heat transfer are seen in the work of Muralidhar and Suzuki [4]. It is expected that wall effects are confined in a layer of $H\sqrt{Da}$ and therefore Da should be very small in the present system.

Under the above assumption the volume averaged, basic temperature T_0 is represented by

$$U_0 \frac{\partial T_0}{\partial X} = \alpha_e \frac{\partial}{\partial Z}\left\{(1+\gamma)\frac{\partial T_0}{\partial Z}\right\} \quad \text{for } Re_d \leqslant 80, \tag{2}$$

where γ is the ratio of the thermal dispersion coefficient $(=\gamma\alpha_e)$ to the effective diffusivity α_e and Z denotes the vertical distance. According to the work of Plumb [5] we set $\gamma = APe_d$, where A is an empirical coefficient. The solution of Eq. (2) is obtained analytically for the present system of isothermal heating:

$$\theta_0 = \sum_{n=0}^{\infty}\left[\operatorname{erf}\left\{\frac{n+1}{\sqrt{(1+\gamma)x}} - \frac{\zeta}{2\sqrt{1+\gamma}}\right\} - \operatorname{erf}\left\{\frac{n}{\sqrt{(1+\gamma)x}} + \frac{\zeta}{2\sqrt{1+\gamma}}\right\}\right]. \tag{3}$$

This equation yields the local Nusselt number Nu_X as

$$Nu_X = 0.564(1+\gamma)^{1/2} Pe_X^{1/2} \quad \text{for } \delta_T < 1, \tag{4}$$

where $\theta_0 = (T_0 - T_i)/\Delta T$, $\zeta = z/\sqrt{x}$ and $(x, z) = (X/Pe_H, Z)/H$. Here $\delta_T (= \Delta_T/H)$ denotes the conventional dimensionless thermal boundary-layer thickness at which $\theta_0 = 0.01$: $\delta_T = 3.64(1+\gamma)^{1/2}x^{1/2}$ for $\delta_T < 1$.

For the present heat transfer system of $Re_d \leqslant 80$, Renken and Poulikakos [6] conducted water experiment with glass spheres of diameter $d = 3$ mm ($Da = 1.16 \times 10^{-6}$) and $H = 7.65$ cm. All of their experimental Nu_X-values were fitted to Eq. (3). The A-value in γ is found to be 5.8×10^{-3} for $Re_d = 1.9$–57.2 and with this A-value the basic temperature profile from Eq. (3) agrees reasonably well with their experimental range of $Pe_X < 10^4$, where the relation of $\delta_T \propto x^{1/2}$ is valid. Fitting of one set of data ($Re_d = 56.6$) at $Pe_X = 1.29 \times 10^5$ produces $A = 5.0 \times 10^{-2}$. With this A-value, Eq. (3) agrees well with experimental data for $0.12 \leqslant z \leqslant 1$ but its deviation is large near the bounding surface. It seems that for $Pe_X > 10^4$ the A-value changes significantly from the surface to the bulk. But Nu_X follows Eq. (4) with $A = 5.8 \times 10^{-3}$. Even though nonuniformities in porosity ε and permeability K exist near the surfaces, a proper choice of γ produces the local temperature profile to represent the actual one to a certain degree. Therefore Eq. (3) is used in the following stability analysis.

3. Propagation theory

Under the linear stability theory the variables are decomposed into the unperturbed quantities and their perturbed ones at the onset position of mixed convection. For this purpose it is assumed that the incipient disturbances will exhibit characteristics of regular longitudinal vortex rolls in the Forchheimer flow. Then the dimensionless disturbance equations are obtained under the Boussinesq approximation as

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{5}$$

$$\frac{1}{Da}\left(1 + Re_K^{-1}\right)v = -\frac{\partial p}{\partial y}, \tag{6}$$

$$\frac{1}{Da}\left(1 + Re_K^{-1}\right)w = -\frac{\partial p}{\partial z} + \theta, \tag{7}$$

$$\frac{\partial \theta}{\partial x} + \left(\frac{Ra}{Re_K}\right) w \frac{\partial \theta_0}{\partial z} = (1+\gamma)\left(\frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right), \tag{8}$$

with the boundary conditions,

$$w = \theta = 0 \text{ at } z = 0 \quad \text{and} \quad z = 1, \tag{9}$$

where

$$(u, v, w) = (U_1/Pe_H, V_1, W_1)Re_K H/\alpha_e,$$

$$y = Y/H \text{ and } p = P_1 H^2/(\rho\alpha_e v).$$

Here ρ is the fluid density and W represents the vertical velocity. The subscript 0 denotes the unperturbed quantity, U_1, V_1 and W_1 are the perturbed velocity components in the Cartesian coordinates and P_1 is the perturbed pressure. The modified Reynolds number $Re_K (= CU_0\sqrt{K}/v)$ represents the inertia effect, where C is the inertia coefficient. It should be noted that the temperature disturbance has been nondimensionalized by $\alpha_e v/(g\beta H^3)$ rather than ΔT. The boundary conditions represent slip on the boundaries and the fixed temperatures on the upper and lower boundaries. The axial heat conduction has been neglected like Eq. (2). Now, we employ the following scaling relation at $x = x_c$, based on Eq. (8):

$$\left(\frac{Ra_{\Delta_T}}{Re_K(1+\gamma)}\right)\frac{w}{\theta} \sim \delta_T^2 \quad \text{for } \delta_T < 1, \tag{10}$$

where Ra_{Δ_T} is the Rayleigh number having the length Δ_T. The order of magnitude of $\Delta T/(\Delta_T(\partial T_0/\partial Z))$ is equivalent to that of $Ra_{\Delta_T}/(Re_K(1+\gamma))$ since the basic temperature profile and its perturbation have been nondimensionalized with different scales.

For $\Delta_T \ll H$, Ra_{Δ_T} is assumed to reach a constant at the critical position X_c [7,8], which leads to $|w/\theta| \sim \delta_T^2$ from Eq. (10). With this scaling relation the perturbed quantities in the form of time-independent, longitudinal vortex rolls are expressed under the normal mode analysis as

$$\begin{bmatrix} v(x,y,z) \\ w(x,y,z) \\ p(x,y,z) \\ \theta(x,y,z) \end{bmatrix} = \begin{bmatrix} (x^{1/2}/a) & v^*(\zeta) \\ x & w^*(\zeta) \\ x^{1/2} & p^*(\zeta) \\ & \theta^*(\zeta) \end{bmatrix} \exp(iay) \quad \text{(11a–d)}$$

with $\delta_T \propto x^{1/2}$,

where i is the imaginary number and a is the wavenumber. The above longitudinal vortex-roll type of flow was observed experimentally for $Pe_H > 0.75$ by Combarnous and Bia [9]. Eqs. (11a–d) are substituted into Eqs. (5)–(8). The resulting equations are functions of ζ only, since Eq. (3) reduces to $\theta_0 = \text{erfc}[\zeta/\sqrt{4(1+\gamma)}]$ for small x. Eliminating v^* and p^* yields the new, self-similar amplitude function:

$$\left[(1+\gamma)(D^2 - a^{*2})\left\{D^2 + \frac{\zeta}{2(1+\gamma)}D - a^{*2}\right\} + \frac{Ra_D^*}{(1+Re_K)}a^{*2}D\theta_0\right]w^* = 0 \quad (12)$$

with the usual boundary conditions of no penetration and isothermal heating,

$$w^* = (D^2 - a^{*2})w^* = 0 \text{ at } \zeta = 0 \quad \text{and} \quad \zeta = 1/x^{1/2}, \quad (13)$$

where $a^* = x^{1/2}a$, $Ra_D^* = x^{1/2}Ra_D$ and $D = d/d\zeta$. For $x \ll 1$, the upper boundary approaches the infinity, i.e. $\zeta \to \infty$. Now, Ra_D^* and a^* are assumed to be the eigenvalues.

The above equations are solved numerically by employing the outward shooting scheme. For a given γ, a^* and Re_K the minimum value of Ra_D^* should be found. In other words, the minimum value of x, i.e. x_c is found for a given Ra_D, Re_K and γ and the corresponding wavenumber is the critical one a_c.

4. Results and conclusion

The critical conditions to mark the onset of regular longitudinal vortex rolls (see Eqs. (11a–d)) have been obtained by using the propagation theory illustrated above. In the deep-pool systems of small x_c the critical values are found to be

$$x_c = \frac{X_c}{HPe_H} = 169(1+\gamma)\left(\frac{1+Re_K}{Ra_D}\right)^2, \quad \text{(14a)}$$

$$a_c = 0.90/\sqrt{x_c(1+\gamma)}, \quad \text{(14b)}$$

which are valid for $\delta_T < 1$, i.e. $x_c < 0.07/(1+\gamma)$, and $\delta_T \propto x^{1/2}$. As expected, x_c increases with increasing γ and Re_K but it decreases with increasing Ra_D. The present system may be said to be stable for $X < X_c$ but it is unstable for $X \geqslant X_c$. The above time-independent, convective instabilities characterized by the present stability criteria represent a fastest growing mode of disturbances for $Pe_H > 0.75$, as mentioned before. For $Pe_H < 0.75$ the time-dependent transverse vortex rolls similar to Tollmien–Schlichting waves will appear with very small U_0 [9].

Now, the stability analysis is extended to the region of $\delta_T \approx 1$ by fixing x locally but varying ζ in Eqs. (3), (11a–d) and (13). Then Eq. (12) can still be kept. It is very interesting that with $\gamma = Re_K = 0$ the results for large x_c approach the well-known critical values of $Ra_{D,c} = 4\pi^2$ and $a_c = \pi$. The resulting overall stability criteria are shown in Fig. 1. For the case of $x_c \to \infty$, i.e. a constant adverse temperature gradient, the term involving ζ in Eq. (12) disappears and the resulting stability criteria are found to be

$$Ra_{D,c} = 4\pi^2(1+\gamma)(1+Re_K) \quad \text{with } a_c = \pi \quad (15)$$

which constitutes the minimum bound.

Considering the available experimental data [2,3,6,9], the present predictions would be valid for $Pe_H > 0.75$ and $Re_d \leqslant 80$ in the system of $\gamma \approx$ constant and also for $Pe_X < 10^4$ in the thermally developing region. With increasing γ and Re_K, the present system becomes more stable. It is interesting that the present stability criteria cover the whole domain of $Ra_D \geqslant 4\pi^2$.

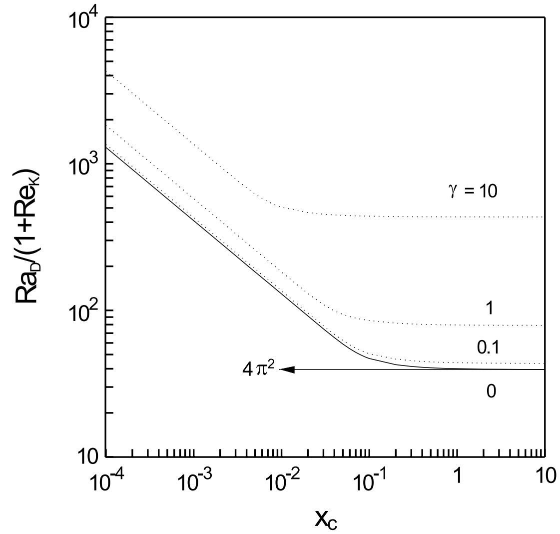

Fig. 1. Prediction of critical position x_c to mark onset of longitudinal vortex rolls.

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